Some notes on the spin-orbit interaction in semiconductor quantum dots

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(1) SO coupling in atoms

Spin of an electron moving (with momentum $\vec{p}$) in an electric field $\vec{E}$ experiences an internal magnetic field proportional to $\vec{E} \times \vec{p}$.

In atoms, the electric field (and thus the internal magnetic field acting on the spin) depends on the orbital - thus the name "spin-orbit effect / interaction / coupling"

$$H_{SO} = -\frac{\hbar}{4m_e^2c^2} \vec{\sigma} \cdot \vec{p} \times (\nabla V)$$
In solids, electrons also experience electric fields from atoms in the lattice. There are two origins for local fields in a crystal:

- (a) Bulk inversion asymmetry (BIA), e.g., zinc-blende structure of GaAs
  => *Dresselhaus spin-orbit interaction*

- (b) Structural inversion asymmetry, (SIA), e.g., heterointerface of GaAs/AlGaAs
  => *Rashba spin-orbit interaction*

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**mag.field acting on spin**

from Hanson *et al.*, RMP **79**, 1217 (2007)
Dresselhaus SO interaction

- Bulk Hamiltonian for SO interaction in zinc-blende crystals (Dyakonov and Perel, 1972):

\[
H^{3D}_{D} \propto \left[ p_x (p_y^2 - p_z^2) \sigma_x + p_y (p_z^2 - p_x^2) \sigma_y + p_z (p_x^2 - p_y^2) \sigma_z \right]
\]

- Going to 2D: integration over the growth direction [typically (001)]:

\[
H^{2D,(001)}_{D} \propto \left[ -p_x \langle p_z^2 \rangle \sigma_x + p_y \langle p_z^2 \rangle \sigma_y + p_x p_z^2 \sigma_x - p_y p_z^2 \sigma_y \right]
\]

- Commonly used linear Dresselhaus term:

\[
H^{2D,001}_{D,\text{linear}} = \beta \left[ -p_x \sigma_x + p_y \sigma_y \right]
\]
Assume that the local electric field points in $z$ direction:

$$H_R \propto \left[ \vec{E} \times \vec{p} \right] \cdot \vec{\sigma} = E_z (-p_y \sigma_x + p_x \sigma_y) = \alpha (-p_y \sigma_x + p_x \sigma_y)$$

where $\alpha$ depends on the material and the confining potential.
If the electron moves (ballistically) some fixed distance, the total spin rotation due to R-SO and **linear** D-SO is independent of the velocity (faster movement => faster rotation)

=> It is useful to define the *spin-orbit length* \( l_{SO} \)
   (typically \( l_{SO} \sim 1 - 10 \mu\text{m} \) in GaAs)

Generally in 2D electron gas (2DEG), the SO couplings lead to spin relaxation view several mechanisms:

- “spin randomization” through random ballistic trajectories (Dyakonov and Perel, 1972)
- spin flipping upon scattering (Elliott, 1954)

Quantum dots: controversy about the importance of the cubic D-SO, generally SO effect is small in GaAs dots, but significant in InAs dots
Some recent results on two-electron quantum dots (GaAs)

\[ H = \sum_{i=1,2} \left[ h(r_i) + h_{SO}(r_i) \right] + \frac{e^2}{4\pi\varepsilon_r\varepsilon_0 |r_1 - r_2|} \]

calculations (exact diagonalization)

experiment (Meunier et al., PRL 98, 126601)

tilted B-field with two angles w.r.t. the planar dot